

G. Bosi: Bi-multi-objective optimization, scalarization and maximal elements of preorders and interval orders

The existence of (weak) Pareto optimal solutions to the classical multi-objective optimization problem is characterized, by referring to the naturally associated preorders and their finite (Richter-Peleg) multi-utility representation.

The case of a compact design space is considered, by using results concerning the existence of maximal elements of preorders. The possibility of reformulating the multi-objective optimization problem for determining the weak Pareto optimal solutions by means of a scalarization procedure is characterized.

Then a generalization of the classical multi-objective optimization to pairs of functions is introduced and discussed. This procedure is referred to as bi-multi-objective optimization.

A justification of this general optimization procedure is presented, related both to multi-objective optimization under ambiguity concerning individual preferences and to Pareto optimality for a family of preferences with nontransitive indifference. Incidentally, the binary relation naturally associated to a bi-multi-objective optimization problem is represented by a finite bi-multi-utility, which generalizes to the nontransitive case the classical finite multi-utility representation. A hopefully significant application is presented to Markowitz portfolio selection under ambiguity concerning both the vector of returns and the covariance matrix.