Local depth and clustering

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Statistical depth functions have been largely used for non-parametric and robust multivariate data analysis. They allow to generalize the concept of median and quantiles to the multivariate setting. Local depth functions have been introduced to also detect local features of the underlying distribution [Agostinelli and Romanazzi, 2011]. These functions involve a tuning parameter and have been investigated, especially in the univariate setting.

I present several results concerning local depth functions for multivariate distributions. In particular, I show consistency and asymptotical normality of the corresponding estimators and derive several analytical results. An interesting feature of local depth functions is their ability to detect the mode of the distribution. Moreover, as the tuning parameter converges to zero, they converge, after an appropriate rescaling, to the underlying density. If the density is differentiable, then so are local depth functions and their derivatives converge to the derivatives of the density. We exploit these results and a gradient system analysis to develop a clustering algorithm based on (i) identification of the modes and (ii) identification of the basins of attraction of the modes via the gradient system. Finally, the finite sample performance of this procedure is illustrated via simulations and data analysis.

This is a joint work with my supervisors Claudio Agostinelli, Alicia Nieto-Reyes and Anand N. Vidyashankar [Francisci et al., 2020].

References

Claudio Agostinelli and Mario Romanazzi. Local depth. *Journal of Statistical Planning and Inference*, 141(2):817–830, 2011.

Giacomo Francisci, Claudio Agostinelli, Alicia Nieto-Reyes, and Anand Vidyashankar. Analytic and statistical properties of local depth motivated by clustering applications. *arXiv preprint arXiv:2008.11957*, 2020.