A. Hamel: From multi-utility representations to complete lattice-valued utility maximization problems (tutorial)

Multi-utility representations seem to be an appropriate way to characterize possibly incomplete preference relations (and some like stochastic dominances or Bewley preferences are introduced in this way). In the last two decades, results of this type were obtained by Dubra, Evren, Maccheroni, Ok, among (and foregoing) many others. However, there is little hope that solving multi-utility maximization problems via multi-objective or vector optimization (see Evren, Ok, JME 2011, as well as Bosi, Herden, JME 2012) will lead to substantial progress. One major structural reason for this unfortunate phenomenon is the fact that infima and suprema with respect to vector orders are basically useless—even if they exist which is very often not the case. This also renders all concepts useless which are based on them (minima/maxima as attained infima/suprema, convex/concave conjugates and duality, infimal/supremal convolution—add your favorite to this list). The question remains how to solve a multi-utility maximization problem.

In this tutorial, it is proposed to skip the vector case and instead transform the multi-utility maximization problem into a problem whose objective maps into a complete lattice of sets generated by a closure operator which in turn is constructed from the multi-utility representation. In this way, solution concepts and existence theorems can be given based on attained suprema, but also involving maximal elements of a certain type. This procedure raises quite a few new mathematical challenges. For example, the complete lattice depends on the multi-utility representation and can be different for different such representations even for the same preference relation (e.g., stochastic dominance orders can be characterized via expected increasing (concave) utilities, (integrated) cdf's, (integrated) quantile functions or even via risk measures). Questions appear like: which one to pick, what information do the corresponding closure operator and the lattice generated by it carry and what relationship does exist among the different closures/lattices for the same preference.

The tutorial gives an introduction into this new approach and discusses a few special cases as well as several new challenges.